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SINGULAR PERTURBATION SOLUTION OF THE PROBLEM VISCOUS FINGERING PHENOMENON IN MULTIFLUID IMMISCIBLE FLOW

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Abstract:
In this paper the phenomenon namely fingering which occurs in the flow problems of oil reservoir engineering has been discussed. The effects arises due to the fingering have been studied by using the Darcy's law together with different kinds of suitable assumptions and conditions. The problem is then modeled into mathematical form which yields second order partial differential equation. The equation is then solved by using singular perturbation technique together with initial and boundary conditions. The solution is then interpreted in terms of fluid flow terms.

Keywords: *Porous; Fingering; Singular; Partial Differential Equation; Similarity; Perturbation.*

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1. Introduction

It is well known that in oil reservoir field the phenomenon of fingering occurs which is caused due to the two immiscible flow in homogeneous porous media. When a fluid existing in the medium is displaced by another fluid of lesser viscosity, instead of regular displacement of the whole front, the fingers may occur which shoots through the porous medium at relatively great speed which is known as the phenomenon of fingering [1]. These fingering depends in particular on the mobility ratio and more likely it is greater than 1. The mobility ratio is defined by $M = \frac{k_w / \mu_w}{k_o / \mu_o}$, where μ_w and μ_o are the viscosities and k_w and k_o are the relative permeability of the displacing (water) and displaced fluids (oil) respectively.

We considered here that a finite cylindrical piece of homogeneous porous medium of length *L*, fully saturated with an oil, which is displaced by injecting fluid water which give rise to fingers. At *x=0,* the initial boundary, it is assumed that complete saturation of water exist due to small displacement of oil. Here *x* is measured in the direction of displacement [2], [3].

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The saturation of water S_w is defined as an average cross-sectional area occupied by the water at level x at time t. Thus $S_w(x,t)$ is the saturation of the water in porous medium represents the average cross-sectional area occupied by fingers [4], [5].

2. MATERIALS AND METHODS

The seepage velocities of water (V_w) and oil (V_o) by assuming the validity of the governing Darcy's law may be written as

$$V_w = -\frac{k_w}{\mu_w} K \frac{\partial p_w}{\partial x} \text{ and } V_o = -\frac{k_o}{\mu_o} K \frac{\partial p_o}{\partial x} \tag{1}$$

Where K is the permeability of the homogeneous medium, p_w and p_o are the pressures of water and oil respectively. Again it is assumed that k_w and k_o are the functions of water saturation (S_w) and oil saturation (S_o) respectively. The equations of continuity are given by (phase densities are considered as constants)

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \text{ and } \phi \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \tag{2}$$

Where ϕ is the porosity of the porous medium. Also by assuming the fully saturation of the porous medium, we may write

$$S_w + S_o = 1 \tag{3}$$

The capillary pressure (p_c) defined as the pressure discontinuity of the flowing fluids across their common interface may be written as

$$p_c = p_o - p_w \tag{4}$$

For fingering phenomenon it is introduced the relationship between the relative permeability, phase saturation and capillary pressure as [6], [7].

$$k_w = S_w,\ k_o = S_o = 1 - S_w,\ p_c = -\beta S_w \tag{5}$$

Here, the negative sign shows the direction of saturation of water is opposite to capillary pressure. Also it is considered β as a small parameter.

The equation of motion for saturation is obtained by substituting the values of V_w and V_o from equation in (1) into the equations in (2).

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$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_w}{\mu_w} K \frac{\partial p_w}{\partial x}\right] \text{and} \quad \phi \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_o}{\mu_o} K \frac{\partial p_o}{\partial x}\right] \tag{6}$$

Eliminating $\frac{\partial p_w}{\partial x}$ from the first equation of (6) using equation (4), we get

$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_w}{\mu_w} K \left\{\frac{\partial p_o}{\partial x} - \frac{\partial p_c}{\partial x}\right\}\right] \tag{7}$$

Combining (7) with the second equation of (6) by using equation (3), we get

$$\frac{\partial}{\partial x}\left[\left\{\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o}\right\} K \frac{\partial p_o}{\partial x} - \frac{k_w}{\mu_w} K \frac{\partial p_c}{\partial x}\right] = 0$$

Integrating above equation with respect to x, we get

$$\left\{\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o}\right\} K \frac{\partial p_o}{\partial x} - \frac{k_w}{\mu_w} K \frac{\partial p_c}{\partial x} = -A \tag{8}$$

For convenience negative sign on the right side is considered with the constant of integration. Here, the value of pressure of oil can be written as [8]

$$p_o = \bar{p} + \frac{p_c}{2}, \quad \bar{p} = \frac{p_c + p_w}{2}. \text{ Which gives } \frac{\partial p_o}{\partial x} = \frac{1}{2}\frac{\partial p_c}{\partial x} \tag{9}$$

Where $\bar{p}$ is the mean pressure, which is constant.

From equations (8) and (9), we get $A = \left\{\frac{k_w}{\mu_w} - \frac{k_o}{\mu_o}\right\} \frac{K}{2} \frac{\partial p_c}{\partial x}$ (10)

Substituting the value of $\frac{\partial p_o}{\partial x}$ from equation (8) into the equation (7) and then substituting the value of A from (10), we get

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[K \frac{k_w}{2\mu_w} \frac{\partial p_c}{\partial x}\right] = 0 \tag{11}$$

Using the conditions in equation (5), we get second order nonlinear partial differential equation for the motion of saturation of water as

$$\phi \frac{\partial S_w}{\partial t} - \frac{\beta K}{4\mu_w} \frac{\partial^2 S_w{}^2}{\partial x^2} = 0 \tag{12}$$

It is small enough to consider β as the perturbation parameter from the assumptions. Also it is the coefficient of the highest derivative in (12), therefore. The problem (12) is a singular perturbation problem. The boundary conditions are may be written as

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$$S_w(0,t) = S_{w0} \text{ and } S_w(L,t) = S_{wl} \quad (13)$$

Due to the assumption that the face x = L is impermeable, there is no flow across this face and thus

$$\frac{\partial}{\partial x} S_w(L,t) = 0 \quad (14)$$

Choosing the dimensionless variables $X = x/L$ and $T = \frac{K}{4\mu_w L^2} t$ the equation (12) together with (13) and (14) becomes

$$\frac{\partial S_w}{\partial T} - \phi \frac{\partial^2 S_w{}^2}{\partial X^2} = 0 \quad (15)$$

$$S_w(0,T) = S_{w0},\ S_w(1,T) = S_{wl},\ \frac{\partial}{\partial X} S_w(1,T) = 0 \quad (16)$$

The equation (15) is converted into nonlinear ordinary differential equation by using the Birkhoff's technique of one parameter group transformation [9].

$$\beta[FF'' + F'^2] + \frac{B}{2}\eta F' - AF = 0, \text{ where } \eta = \frac{X}{T^B}, F(\eta) = \frac{S_w(X,T)}{T^{2A}}, B = \frac{1+2A}{2} \quad (17)$$

The corresponding conditions can be written as

$$F(0) = S_{w0}, F(\alpha) = S_{wl} \text{ and } F'(\alpha) = 0 \quad (18)$$

The equation (17) together with (18) has been solved by using singular perturbation technique, called the method of composite expansions.

Since the coefficient of F' is greater than zero in $(0,\alpha)$, the nonuniformity occurs near $\eta = 0$. To describe the region of non-uniformity, we need a stretching transformation $\varsigma = \eta/\beta$. We determine an expansion for F by using the technique of Bromberg, Visik and Lyusternik [10]. Assuming the expansion

$$F(\eta,\beta) = H(\eta,\beta) + G(\varsigma,\beta) = H_0(\eta) + G_0(\varsigma) + \beta[H_1(\eta) + G_1(\varsigma)] + \ldots \quad (19)$$

After considering the inner and outer expansions we obtain the following values

$$H_0(\eta) = S_{wl} T^{2A} \eta^{2A/B}, \quad G_0(\varsigma) = S_{w0}, \quad H_1(\eta) = -2S_{wl}{}^2 \frac{A(6A-1)}{B^2} T^{4A+1} \eta^{2A/B} \left\{1 - \frac{1}{T\eta^{1/b}}\right\},$$

$$G_1(\varsigma) = \frac{A}{2}\varsigma^2 - \left\{\frac{2A}{B\alpha} S_{wl} + A\alpha\beta^{-1}\varsigma\right\}$$

Substituting these values in (19), we get

$$F(\eta,\beta) = S_{wl} T^{2A} \eta^{2A/B} + S_{w0} - 2S_{wl}{}^2 \frac{A(6A-1)}{B^2} T^{4A+1} \eta^{2A/B} \left\{1 - \frac{1}{T\eta^{1/b}}\right\} + \frac{A}{2}\varsigma^2 - \left\{\frac{2A}{B\alpha} S_{wl} + A\alpha\beta^{-1}\varsigma\right\} + \ldots$$

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Hence the required solution for water saturation is obtained in the form $S_w(X,T) = T^{2A} F\left\{\frac{X}{T^B}\right\}$, where $F\left\{\frac{X}{T^B}\right\}$ can be obtained from $F(\eta, \beta)$ by substituting $\varsigma = \eta / \beta$ and $\eta = X / T^B$.

3. Conclusions & Recommendations

The result obtained for the phenomenon of fingering is the higher ordered approximate solution for the distribution of the saturation of the displacing fluid water. This is the analytical expression of the average cross-sectional area occupied by the fingers. Due to the analytic approach only the graphs have not been obtained in this paper.

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