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INVARIANCE UNDER CORDIALITY OF PATH UNION OF $C_5 \odot P_K$

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Abstract:
We obtain different structures of $P_m(G)$ for $G = C_5 \odot P_k$.We take k=2,3,4,5,6.The different structures are obtained on path union because of we use different vertices on $C_5 \odot P_k$ to construct $P_m(G)$. We show all structures so obtained are cordial. Hence invariance under cordiality**.**

Keywords: *Invariance; Cordial Labeling; Path Union; Path; Cycle.*

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1. Introduction

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [6], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West. [9]. I.Cahit introduced the concept of cordial labeling [5]. F: V (G) $\rightarrow \{0, 1\}$ is a function. From this label of any edge (uv) is given by $|f(u)-f(v)|$. Further number of vertices labeled with 0 i.e$v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e. $e_f(0)$ and number of edges labeled with 1 i.e. $e_f(1)$ differ at most by one. Then the function f is called as cordial labeling. I.Cahit has shown that: every tree is cordial; K_n is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4). A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [8].

Our focus of attention is on path unions on different graphs. For a given graph there are different path unions (up to isomorphism) structures possible. It depends on which point on G is used to fuse with vertex of Pm to obtain path-union. We have shown that for G = bull on C_3, bull on C_4, C_3^+, C_4^+-e then different path union $P_m(G)$ are cordial [4]. It is called as invariance under cordial labeling. We use the convention that $v_f(0,1) = (a,b)$ to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b in number. Further $e_f(0,1) = (x,y)$ we mean the number of edges labeled with 0 are x and number of edges labeled with 1 are y in number. The graph whose cordial labeling is known is cordial graph.

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2. Preliminaries

Fusion of vertex Let G be a (p,q) graph. Let$u \neq v$ be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges. [9] . **Path union of G** ,i.e.$P_m(G)$ is obtained by taking a path p_m and take m copies of graph G Then fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges. Where G is a (p,q) graph.

3. Theorems Proved

3.1. Theorem: All structures of P_m (G) are cordial for G $=C_5 \odot P_2$

Proof: Define f: V $(P_m(G)) \rightarrow \{0,1\}$ as follows. f gives two labeled copies of $C_5 \odot P_2$ Type A and Type B. These are shown in fig 5.1 and fig 5.2 below.

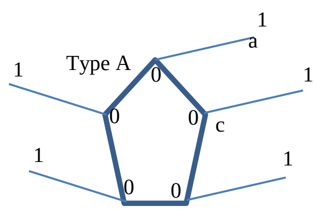

Fig 5.1 $v_f(0,1)=(5,5)$, $e_f(0,1)$ =

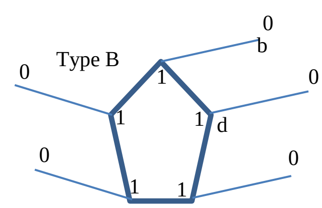

Fig 5.2 $v_f(0,1)=(5,5)$, $e_f(0,1)$ = (5,5)

There are two structures possible on $P_m(G)$. In **structure1** we choose vertex 'a' from type A and 'b' from Type B to fuse on vertex of path $P_m = (v_1, v_2, v_3, \ldots v_m)$. In **structure2** we choose vertex 'c' from type A and vertex 'd' from Type B to fuse on vertex of path P_m. Note that any of the 5 pendent vertices each from Type A and that from Type B will be vertex 'a' or vertex 'b' respectively. And any of the 5 degree three vertices from each type A and Type B will be vertex 'c'and 'd' respectively.

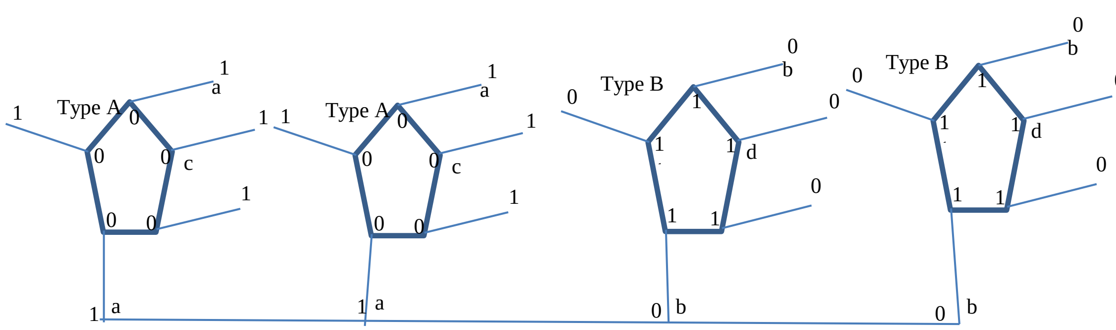

Fig 5.3 Labeled copy of P4($C_5 \odot P_2$): $v_f(0,1)=(20,20)$, $e_f(0,1)$ = (22,21); structure1

Structure 1: We start with a unlabeled path P_m and at vertex v_i on it fuse vertex 'a' from type A if $i \equiv 1, 2 \pmod 4$ and vertex 'b' from type b if $i \equiv 2,3 \pmod 4$.The label of edge (aa) =label of edge (bb)= 0 and label of edge (ab) = 1. The label numbers in this case are $v_f(0,1)=(5m,5m)$ for all m

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and for edges when m = 2x, x= 1,2,.. we have $e_f(0,1)$ = (5m+x,5m+x-1) and when m is of type 2x+1, x= 0,1, 2 …we have $e_f(0,1)$= (5m+x,5m+x).

For **structure2** we repeat the same procedure as above. We start with a unlabeled path P_m and at vertex v_i on it fuse vertex 'c' from type A if i≡1, 2(mod 4) and vertex 'd' from type b if i≡2,3 (mod 4).The label of edge (cc) =label of edge (dd)= 0 and label of edge (cd) = 1. The label numbers in this case are $v_f(0,1)$=(5m,5m) for all m and for edges when m = 2x, x= 1,2,.. we have $e_f(0,1)$ = (5m+x,5m+x-1) and when m is of type 2x+1, x= 0,1, 2 …we have $e_f(0,1)$= (5m+x,5m+x). The graph is cordial. #.

3.2. Theorem: All structures of $P_m(G)$ are cordial for G $=C_5 \odot P_3$.

Proof: Define f: $V(P_m(G)) \rightarrow \{0,1\}$ as follows. f gives two labeled copies of $C_5 \odot P_2$ Type A and Type B. These are shown in fig 5.3and fig 5.4 below.

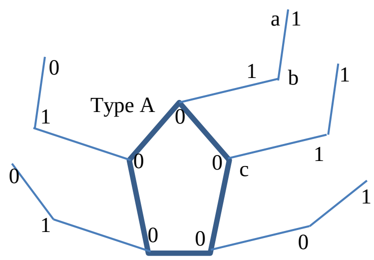

Fig 5.4 $v_f(0,1)$=(8,7), $e_f(0,1)$ = (8 ,7)

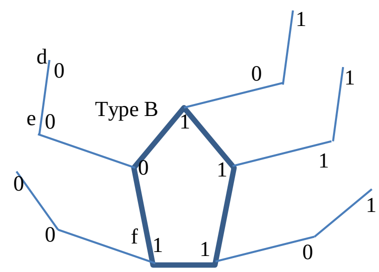

Fig 5.5 $v_f(0,1)$=(7,8), $e_f(0,1)$ = (8,7)

Different structures on path union are due to we use different vertices on G namely 'a', 'b' and 'c' to fuse with vertex of $P_m = (v_1, v_2, ..v_m)$

In **structure 1** we fuse vertex 'a' from Type A with vertex v_i of P_m and vertex 'd' from Type B with vertex v_i of $P_m$alternately starting with type A. On Pm the label of edge (ad) =1 and label(da)=1.The label number distribution on resultant graph are $v_f(0,1)$=(8+15x,7+15x) when m is oddnumber given by 2x+1,x= 0, 1, 2.. , And$v_f(0,1)$=(15x,15x) when m is even number given by 2x, x= 1, 2, .. For all m $e_f(0,1)$ = (8m, 8m-1).

In **structure 2** we fuse vertex 'b' from Type A with vertex v_i of P_m and vertex 'e' from Type B with vertex v_i of P_m alternately starting with type A. On Pm the label of edge (be) =1 and label(eb) =1.The label number distribution on resultant graph are $v_f(0,1)$=(8+15x,7+15x) when m is odd number given by 2x+1,x= 0, 1, 2.. , And $v_f(0,1)$=(15x,15x) when m is even number given by 2x, x= 1, 2, .. For all m $e_f(0,1)$ = (8m, 8m-1).

In **structure 3** we fuse vertex 'c' from Type A with vertex v_i of P_m and vertex 'f' from Type B with vertex v_i of P_m alternately starting with type A. On Pm the label of edge (be) =1 and label(eb) =1.The label number distribution on resultant graph are $v_f(0,1)$=(8+15x,7+15x) when m is odd number given by 2x+1,x= 0, 1, 2.. , And $v_f(0,1)$=(15x,15x) when m is even number given by 2x, x= 1, 2, .. For all m $e_f(0,1)$ = (8m, 8m-1).Thus the graph is cordial on all of it's structures.

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3.3. Theorem: All structures of $P_m(G)$ are cordial for $G = C_5 \odot P_4$

Proof: Define f: $V(P_m(G)) \rightarrow \{0,1\}$ as follows. f gives two labeled copies of $C_5 \odot P_4$ Type A and Type B. These are shown in fig 5.6 and fig 5.7 below. They have equal label numbers but different distribution.

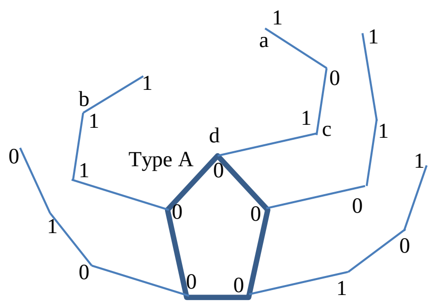

Fig 5.6 $v_f(0,1)=(10,10)$, $e_f(0,1) = (10,10)$

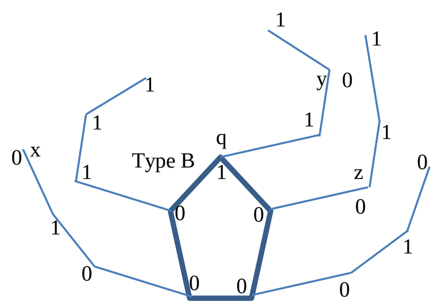

Fig 5.7 $v_f(0,1)=(10,10)$, $e_f(0,1) = (10,10)$

In **structure 1** we fuse vertex ‘a’ from Type A with vertex v_i of P_m if i≡ 1, 2(mod 4) and vertex ‘x’ from Type B with vertex v_i of P_m if i≡ 2,3(mod 4). On P_m the label of edge (ax) =1 and label(xa) =1,label of edge (xx) =0 and label(aa) =0.The label number distribution on resultant graph are $v_f(0,1)=(10m,10m)$for all m and when m is odd number given by 2x+1,x= 0, 1, 2.. , And $v_f(0,1)=(10m+x,10m+x)$ when m is even number given by 2x, x= 1, 2, .. for all m $e_f(0,1) = (10m, 10m-1)$.

In **structure 2** we fuse vertex ‘b’ from Type A with vertex v_i of P_m if i≡ 1, 2(mod 4) and vertex ‘y from Type B with vertex v_i of P_m if i≡ 2,3 (mod 4). On P_m the label of edge (by) =1 and label(yb) =1,label of edge (yy) =0 and label(bb) =0.The label number distribution on resultant graph are $v_f(0,1)=(10m,10m)$ for all m and when m is odd number given by 2x+1,x= 0, 1, 2.. , And $v_f(0,1)=(10m+x,10m+x)$ when m is even number given by 2x, x= 1, 2, .. for all m $e_f(0,1) = (10m, 10m-1)$.

In **structure 3** we fuse vertex ‘c’ from Type A with vertex v_i of P_m if i≡ 1,2(mod 4) and vertex ‘z from Type B with vertex v_i of P_m if i≡ 2,3 (mod 4). On P_m the label of edge (cz) =1 and label(xc) =1,label of edge (zz) =0 and label(cc) =0.The label number distribution on resultant graph are $v_f(0,1)=(10m,10m)$ for all m and when m is odd number given by 2x+1,x= 0, 1, 2.. , And $v_f(0,1)=(10m+x,10m+x)$ when m is even number given by 2x, x= 1, 2, .. for all m $e_f(0,1) = (10m, 10m-1)$.

In **structure 4** we fuse vertex ‘q’ from Type B with vertex v_i of P_m if i≡ 1, 2(mod 4) and vertex ‘d from Type A with vertex v_i of P_m if i≡ 2,3 (mod 4). On P_m the label of edge (qd) =1 and label(dq) =1,label of edge (qq) =0 and label(dd) =0.The label number distribution on resultant graph are $v_f(0,1)=(10m,10m)$ for all m and when m is odd number given by 2x+1,x= 0, 1, 2.. . And $v_f(0,1)=(10m+x,10m+x)$ when m is even number given by 2x, x= 1, 2, .. for all m, $e_f(0,1) = (10m, 10m-1)$. The resultant graph structures of $P_m(G)$ are cordial.

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3.4. Theorem: All Structures of $P_m(G)$ Are Cordial for $G = C_5 \odot P_5$.

Proof: Define f: $V(P_m(G)) \rightarrow \{0,1\}$ as follows. f gives two labeled copies of $C_5 \odot P_2$ Type A and Type B. These are shown in fig 5.8 and fig 5.9below.

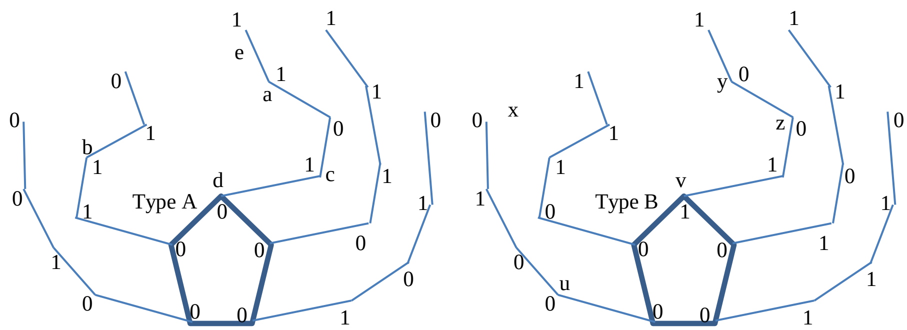

Fig 5.8 $v_f(0,1)=(13,12)$, $e_f(0,1) = (13,12)$ Fig 5.9 $v_f(0,1)=(12,13)$, $e_f(0,1) = (13,12)$

Different structures on path union are due to we use different vertices on G namely 'e', 'a', 'b', 'c', 'd' to fuse with vertex of $P_m = (v_1, v_2, .. v_m)$.

In **structure 1** we fuse vertex 'e' from Type A with vertex v_i of P_m and vertex 'x' from Type B with vertex v_i of P_m alternately starting with type A. On P_m the label of edge (ex) =1 and label(xe) =1, label of edge (ee) =0 and label(xx) =0.The label number distribution on resultant graph are $v_f(0,1)=(13+25x,12+25x)$ when m is odd number given by 2x+1,x= 0, 1, 2.. , And $v_f(0,1)=(25x,25x)$ when m is even number given by 2x, x= 1, 2, .. For all m $e_f(0,1) = (13m, 13m-1)$.

In **structure 2** we fuse vertex 'a' from Type A with vertex v_i of P_m and vertex 'y' from Type B with vertex v_i of P_m alternately starting with type A. On P_m the label of edge (ay) =1 and label(ya) =1, label of edge (aa) =0 and label(yy) =0.The label number distribution on resultant graph are $v_f(0,1)=(13+25x,12+25x)$ when m is odd number given by 2x+1,x= 0, 1, 2.. , And $v_f(0,1)=(25x,25x)$ when m is even number given by 2x, x= 1, 2, .. For all m $e_f(0,1) = (13m, 13m-1)$.

In **structure 3** we fuse vertex 'b' from Type A with vertex v_i of P_m and vertex 'z' from Type B with vertex v_i of P_m alternately starting with type A. On P_m the label of edge (bz) =1 and label(zb) =1, label of edge (zz) =0 and label(bb) =0.The label number distribution on resultant graph are $v_f(0,1)=(13+25x,12+25x)$ when m is odd number given by 2x+1,x= 0, 1, 2.. , And $v_f(0,1)=(25x,25x)$ when m is even number given by 2x, x= 1, 2, .. For all m,$e_f(0,1) = (13m,13m-1)$.

In **structure 4** we fuse vertex 'c' from Type A with vertex v_i of $P_m$and vertex 'u' from Type B with vertex v_i of P_m alternately starting with type A. On P_m the label of edge (cu) =1 and label(uc) =1, label of edge (uu) =0 and label(cc) =0.The label number distribution on resultant graph are $v_f(0,1)=(13+25x,12+25x)$ when m is odd number given by 2x+1,x= 0, 1, 2.. , And $v_f(0,1)=(25x,25x)$ when m is even number given by 2x, x= 1, 2, .. For all m $e_f(0,1) = (13m, 13m-1)$.

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In **structure 5** we fuse vertex'd' from Type A with vertex v_i of P_m and vertex 'v' from Type B with vertex v_i of P_m alternately starting with type A. On P_m the label of edge (dv) =1 and label(vd) =1, label of edge (dd) =0 and label(vv) =0.The label number distribution on resultant graph are $v_f(0,1)=(13+25x,12+25x)$ when m is odd number given by 2x+1,x= 0, 1, 2.. , And $v_f(0,1)=(25x,25x)$ when m is even number given by 2x, x= 1, 2, .. For all m $e_f(0,1) = (13m, 13m-1)$.

3.5. Theorem: All structures of $P_m(G)$ are cordial for G $=C_5\odot P_6$.

Proof: Define f: $V(P_m(G))\rightarrow\{0,1\}$ as follows. f gives a labeled copy of $C_5\odot P_2$namely Type A as shown below.We use it repeatedly in a specific way as shown below to obtain labeled copy of G.

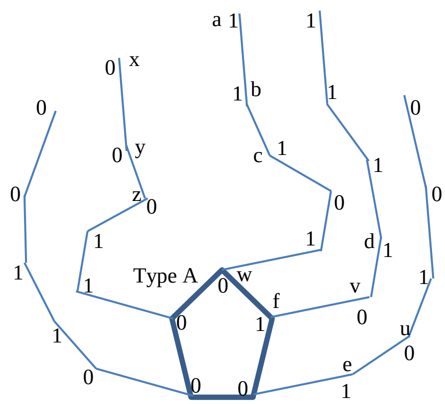

Fig 5.8 $v_f(0,1)=(15,15)$, $e_f(0,1) = (15 ,15)$

We get 6 different structures on path union depending on the point on C_5□P_6 fused with vertex of path P_m to obtain $P_m(G)$.We start with a unlabeled copy of P_m.

Structure1: At vertex v_i of P_m fuse a labeled copy as above (type A) at point 'a' on it for i≡ 1,2 (mod 4) and at vertex 'x' if i≡ 0,3 (mod 4). The label number distribution is $v_f(0,1)=(15m,15m)$ for all m , and on edges when m = 2x+1, x= 0, 1, .. .$e_f(0,1) = (15+x ,15+x)$ and if m =2x, x=1, 2, .. $e_f(0,1) = (15+x ,15+x-1)$.

Structure2: At vertex v_i of P_m fuse a labeled copy as above (type A) at point 'b' on it for i≡ 1,2 (mod 4) and at vertex 'y' if i≡ 0,3 (mod 4). The label number distribution is $v_f(0,1)=(15m,15m)$ for all m , and on edges when m = 2x+1, x= 0, 1, .. .$e_f(0,1) = (15+x ,15+x)$ and if m =2x, x=1, 2, .. $e_f(0,1) = (15+x ,15+x-1)$.

Structure3: At vertex v_i of P_m fuse a labeled copy as above (type A) at point 'c' on it for i≡ 1,2 (mod 4) and at vertex 'z' if i≡ 0,3 (mod 4). The label number distribution is $v_f(0,1)=(15m,15m)$ for all m , and on edges when m = 2x+1, x= 0, 1, .. .$e_f(0,1) = (15+x ,15+x)$ and if m =2x, x=1, 2, .. $e_f(0,1) = (15+x ,15+x-1)$.

Structure4: At vertex v_i of P_m fuse a labeled copy as above (type A) at point 'd' on it for i≡ 1,2 (mod 4) and at vertex 'u' if i≡ 0,3 (mod 4). The label number distribution is $v_f(0,1)=(15m,15m)$ for all m , and on edges when m = 2x+1, x= 0, 1, .. .$e_f(0,1) = (15+x ,15+x)$ and if m =2x, x=1, 2, .. $e_f(0,1) = (15+x ,15+x-1)$.

Structure5: At vertex v_i of P_m fuse a labeled copy as above (type A) at point 'e' on it for i≡ 1,2 (mod 4) and at vertex 'v' if i≡ 0,3 (mod 4). The label number distribution is $v_f(0,1)=(15m,15m)$

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for all m , and on edges when m = 2x+1, x= 0, 1, .. .$e_f(0,1)$ = (15+x ,15+x) and if m =2x, x=1, 2, .. $e_f(0,1)$ = (15+x ,15+x-1).

Structure6: At vertex v_i of P_m fuse a labeled copy as above (type A) at point 'f' on it for i≡ 1,2 (mod 4) and at vertex 'w' if i≡ 0,3 (mod 4). The label number distribution is $v_f(0,1)$=(15m,15m) for all m , and on edges when m = 2x+1, x= 0, 1, .. .$e_f(0,1)$ = (15+x ,15+x) and if m =2x, x=1, 2, .. $e_f(0,1)$ = (15+x ,15+x-1).Thus the graph is cordial for all non-isomorphic structures on path union.

4. Conclusions

To obtain a path union we generally fuse a copy of G ata fixed point on G with every vertex of P_m.We have considered every vertex of $C_5 \odot P_k$ that will produce non-isomorphic structure on path union.We take k =2, 3, 4, 5 and 6.We take all path unions and show that all are cordial. This is also called as invariance under cordiality of path union.

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