 ISSN: 2454-1907
DOI: 10.5281/zenodo.1149113

IJETMR
International Journal of Engineering Technologies and Management Research
A Knowledge Repository

EVOLUTION OF MODULATIONAL INSTABILITY IN TRAVELLING WAVE SOLUTION OF NON-LINEAR PARTIAL DIFFERENTIAL EQUATION

Ram Dayal Pankaj [*1], Arun Kumar [2], Chandrawati Sindhi [1]
[*1]Department of Mathematics, J.N.V. University, Jodhpur (Rajasthan), India
[2] Department of Mathematics, Government College, Kota (Rajasthan), India

Abstract:
The Ritz variational method has been applied to the nonlinear partial differential equation to construct a model for travelling wave solution. The spatially periodic trial function was chosen in the form of combination of Jacobian Elliptic functions, with the dependence of its parameters.

Keywords: *Jacobi Elliptic Functions; Ritz Variational Method; Spatially Periodic Trial Function.*

Cite This Article: Ram Dayal Pankaj, Arun Kumar, and Chandrawati Sindhi. (2018). "A SUGGESTION TO EVALUATION OF CIVIL ARCHITECTURE: BALIKESIR EXAMPLE." *International Journal of Engineering Technologies and Management Research,* 5(1), 1-7. DOI: 10.5281/zenodo.1149113.

1. Introduction

To find exact solutions of Nonlinear Partial Differential Equations (PDEs) is an important but difficult task. Now, many methods have been established to find exact solutions of PDEs especially for integrable systems. Moreover, it is clear that different methods usually give different types of exact solutions. In some cases, similar classes of solutions can be obtained by several approaches. Recently, the multi-linear variable separation approach has been proved very powerful in finding variable separation solutions for a large number of PDEs.

2. Method for Nonlinear Partial Differential Equations

The Nonlinear Partial Differential Equations (NLPDE)

$$iE_t + \beta_1 E_{xx} + i\gamma_1 E_{xxx} + \delta_1 |E|^2 E + 3i\alpha_1 |E|^2 E_x = 0 \tag{1}$$

Here $E\ (x, t)$ is high-frequency field and $\alpha_1, \beta_1, \gamma_1, \delta_1$ are real constant with the relation $\begin{vmatrix} \alpha_1 & \gamma_1 \\ \delta_1 & \beta_1 \end{vmatrix} =$ constant. Where the cubic term in Eq. (1) describes the nonlinear-self interaction in the high frequency subsystem, such a term corresponds to a self-focusing effect in plasma physics. In this

ISSN: 2454-1907
DOI: 10.5281/zenodo.1149113

NLPDE with, one of limit $\alpha_1 = \gamma_1 = 0$, reduces to non linear Schrodinger equation which describes a plane self-focusing and one dimensional self-modulation of waves in nonlinear dispersive media [14]. In another limit, $\beta_1 = \delta_1 = 0$ Equation reduces to the modified Korteweg-de Vries equation. If we take $\beta_1 = \gamma_1 = 1, \delta_1 = \alpha_1 = -\frac{1}{3}$ then NLPDE reduces to Hirota equation [16].

The equation (1) can be formulated as a variational problem corresponding to the Lagrangian

$$L(t) = \int_{-\lambda/2}^{\lambda/2} \Re(x,t)dx \tag{2}$$

The Lagrangian density $\Re(x,t)$ is given by

$$\Re(x,t) = \frac{i}{2}\left[E^* E_t - EE_t^*\right] - \beta_1 |E_x|^2 - i\gamma_1 E_{xx}E_x^* + \frac{3i\alpha_1}{2}|E|^2 E^* E_x + \frac{\delta_1}{2}\left(E^* E\right)^2 \tag{3}$$

The star denotes the complex conjugate and the limits of the integration is the Periodicity length λ for the solutions.
We employ the Ritz optimization procedure to the action integral

$$S(t) = \int L(t)dt \tag{4}$$

With respect to time dependent parameters of a trial function which admits
(1) The shape of an unmodulated wave with a small sinusoidal disturbance.
(2) Provide spatial periodicity of the Lagrangian with period λ.

So these features are provided by

$$E(x,t) = A(t)\left[dn(z;\beta) + \beta cn(z;\beta)\right]\exp\left[i\left\{\frac{kz}{\alpha} + c\cos\left(\frac{2\pi z}{\alpha\lambda}\right) + \phi\right\}\right] \tag{5}$$

The time-dependent functions, independent from one another, are $A, \beta, c, \varphi, \alpha, x_0$ and k. Here, $dn(z;\beta)$ and $cn(z;\beta)$ are the Jacobian elliptic functions for $z = \alpha(x - x_0)$ and $\alpha = 4K(\beta)/\lambda$, and $K(\beta)$ is the complete elliptic integral of first kind. For small parameters B and c, one obtains from equation (5) the following expressions:

$$E(x,t) = A(t)\left[1 + (B + ic)\cos z\right]\exp\left[i\left\{\frac{kz}{\alpha} + \phi\right\}\right] \tag{6}$$

ISSN: 2454-1907
DOI: 10.5281/zenodo.1149113

Equation (6) describes envelopes of a finite amplitude wave (with wave number *k*), slightly modulated by a plane wave with wave number α .Now we proceed to study the variation equations for the parameters A, β c and ϕ with the help of Euler-Lagrange equations. Upon substituting the trial function (6) in (3) with (2), Put $x_{0t} = V$, *k (t) = V/2* (constant).

So the L (t) become

$$
\begin{aligned}
L(t) &= A^2\left[\alpha^{-1}\left(\frac{Vx_{0t}}{2}-\frac{\beta_1 V^2}{4}-\phi_t\right)+\frac{\gamma_1 V^3}{8}\right]I_2 + A^2\alpha^{-1}\left(-c_t+\frac{2\pi^2 i\gamma_1 Vc}{\lambda^2}\right)I_1 + \frac{2A^2\pi^2c^2}{\lambda^2}\left(3V\gamma_1 - 2\beta_1\alpha^{-1}\right)I_3 \\
&+ A^2\alpha(\gamma_1 V-\beta_1)I_4 + A^4\alpha^{-1}\left(\frac{\delta_1}{2}-\frac{3\alpha_1 V}{2}\right)I_5 + \frac{3A^4\pi c\alpha_1\alpha^{-1}}{\lambda}I_9 + 3i\alpha A^4 I_{10} - i\gamma_1 A^2\alpha^2 I_{11} - \frac{\gamma_1 A^2\alpha V}{2}I_{12} \\
&+ \frac{i\gamma_1 V^2 A^2\alpha}{4}I_{13} - \frac{4A^2\alpha\pi c\gamma_1}{\lambda}I_{18} + \frac{2\pi c\gamma_1\alpha A^2}{\lambda}I_{17} - \frac{8\pi^3c^3A^2\gamma_1}{\lambda^3}I_6 - \frac{8A^2 i\gamma_1\alpha^{-1}\pi^2 c}{\lambda^2}I_8 - \frac{4\pi^2 c\gamma_1 A^2}{\lambda^2}I_{15} \\
&- (2+\alpha V)\frac{2i\pi c\gamma_1 A^2}{\lambda}I_{14} + \frac{4\pi^2c^2 i\gamma_1\alpha A^2}{\lambda^2}I_{16}
\end{aligned}
\tag{7}
$$

$$
I_1 = \int_{-2k}^{2k}\left[dn(z;\beta)+\beta cn(z;\beta)\right]^2\cos(2\pi z/\alpha\lambda)dz = \frac{2\pi^2\,\mathrm{sgn}(\beta)}{K\sinh\left(\frac{\pi K'}{2K}\right)} \tag{8}
$$

$$
I_2 = \int_{-2k}^{2k}\left\{dn(z;\beta)+\beta cn(z;\beta)\right\}^2 dz = 4c_1 \tag{9}
$$

$$
I_3 = \int_{-2k}^{2k}\left\{dn(z;\beta)+\beta cn(z;\beta)\right\}^2\sin^2\left(2\pi z/\alpha\lambda\right)dz = 2\left[c_1 - \frac{\pi^2}{K\sinh\left(\frac{\pi K'}{2K}\right)}\right] \tag{10}
$$

$$
I_4 = \int_{-2k}^{2k}\left[\frac{d}{dz}\left\{dn(z;\beta)+\beta cn(z;\beta)\right\}\right]^2 dz = \frac{4c_2}{3} \tag{11}
$$

$$
I_5 = \int_{-2k}^{2k}\left\{dn(z;\beta)+\beta cn(z;\beta)\right\}^4 dz = \frac{4c_3}{3} \tag{12}
$$

$$
I_6 = \int_{-2k}^{2k}\left[dn(z;\beta)+\beta cn(z;\beta)\right]^2\sin^3(2\pi z/\alpha\lambda)dz = 0 \tag{13}
$$

$$
I_7 = \int_{-2k}^{2k}\left[dn(z;\beta)+\beta cn(z;\beta)\right]^2\sin(2\pi z/\alpha\lambda)dz = 0 \tag{14}
$$

 ISSN: 2454-1907
DOI: 10.5281/zenodo.1149113

$$I_8 = \int_{-2k}^{2k} [dn(z;\beta) + \beta cn(z;\beta)]^2 \cos(2\pi z/\alpha\lambda)\sin(2\pi z/\alpha\lambda) dz = 0 \tag{15}$$

$$I_9 = \int_{-2k}^{2k} [dn(z;\beta) + \beta cn(z;\beta)]^4 \sin(2\pi z/\alpha\lambda) dz = 0 \tag{16}$$

$$\begin{aligned} I_{10} &= \int_{-2k}^{2k} \{dn(z;\beta) + \beta cn(z;\beta)\}^3 \frac{d}{dz}\{dn(z;\beta) + \beta cn(z;\beta)\} dz \\ &= \frac{1}{4}\int_{-2k}^{2k} \frac{d}{dz}\{dn(z;\beta) + \beta cn(z;\beta)\}^4 dz = \frac{1}{4}\left[\{dn(z;\beta) + \beta cn(z;\beta)\}^4\right]_{-2k}^{2k} = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} I_{11} &= \int_{-2k}^{2k} \frac{d}{dz}\{dn(z;\beta) + \beta cn(z;\beta)\}\frac{d^2}{dz^2}\{dn(z;\beta) + \beta cn(z;\beta)\} dz \\ &= \frac{1}{2}\int_{-2k}^{2k} \frac{d}{dz}\left[\frac{d}{dz}\{dn(z;\beta) + \beta cn(z;\beta)\}\right]^2 dz = \frac{1}{2}\left[\left[\frac{d}{dz}\{dn(z;\beta) + \beta cn(z;\beta)\}\right]^2\right]_{-2k}^{2k} = 0 \end{aligned} \tag{18}$$

$$\begin{aligned} I_{12} &= \int_{-2k}^{2k} \{dn(z;\beta) + \beta cn(z;\beta)\}\frac{d^2}{dz^2}\{dn(z;\beta) + \beta cn(z;\beta)\} dz \\ &= \left[\{dn(z;\beta) + \beta cn(z;\beta)\}\frac{d}{dz}\{dn(z;\beta) + \beta cn(z;\beta)\}\right]_{-2k}^{2k} - \int_{-2k}^{2k}\left[\frac{d}{dz}\{dn(z;\beta) + \beta cn(z;\beta)\}\right]^2 dz \\ &= \frac{1}{2}\left[\frac{d}{dz}(dn + \beta cn)^2\right]_{-2K}^{2K} - I_4 = -\frac{4c_2}{3} \end{aligned} \tag{19}$$

$$\begin{aligned} I_{13} &= \int_{-2k}^{2k} \{dn(z;\beta) + \beta cn(z;\beta)\}\frac{d}{dz}\{dn(z;\beta) + \beta cn(z;\beta)\} dz \\ &= \frac{1}{2}\int_{-2k}^{2k} \frac{d}{dz}\{dn(z;\beta) + \beta cn(z;\beta)\}^2 dz = \frac{1}{2}\left[(dn + \beta cn)^2\right]_{-2k}^{2k} = 0 \end{aligned} \tag{20}$$

$$I_{14} = \int_{-2k}^{2k} \{dn(z;\beta) + \beta cn(z;\beta)\}\frac{d}{dz}\{dn(z;\beta) + \beta cn(z;\beta)\}\sin(2\pi z/\alpha\lambda) dz = 0 \tag{21}$$

$$I_{15} = \int_{-2k}^{2k} \{dn(z;\beta) + \beta cn(z;\beta)\}\frac{d}{dz}\{dn(z;\beta) + \beta cn(z;\beta)\}\cos(2\pi z/\alpha\lambda) dz = 0 \tag{22}$$

 ISSN: 2454-1907
DOI: 10.5281/zenodo.1149113

$$I_{16} = \int_{-2k}^{2k} \{dn(z;\beta) + \beta cn(z;\beta)\} \frac{d}{dz} \{dn(z;\beta) + \beta cn(z;\beta)\} \sin^2(2\pi z/\alpha\lambda) dz = 0 \tag{23}$$

$$I_{17} = \int_{-2k}^{2k} \{dn(z;\beta) + \beta cn(z;\beta)\} \frac{d^2}{dz^2} \{dn(z;\beta) + \beta cn(z;\beta)\} \sin(2\pi z/\alpha\lambda) dz = 0 \tag{24}$$

$$I_{18} = \int_{-2k}^{2k} \left[\frac{d}{dz} \{dn(z;\beta) + \beta cn(z;\beta)\} \right]^2 \sin(2\pi z/\alpha\lambda) dz = 0 \tag{25}$$

Here $K'(\beta) = K\left[(1-\beta^2)^{-\frac{1}{2}}\right]$ and c_1 , c_2 , c_3 are the following combinations of elliptic integrals *K*, *E* and their argument β :

$$c_1 = 2E - (1-\beta^2)K$$
$$c_2 = (1+\beta^2)E - (1-\beta^2)K$$
$$c_3 = 8(1+\beta^2)E - (5+3\beta^2)(1-\beta^2)K$$

Put (8) to (25) in equation (7), we get

$$L(t) = \frac{A^2 c_1 \lambda}{K} A^2 c_1 \left(\frac{V}{2} x_{0t} - \frac{\beta_1 V^2}{4} - \phi_t + \frac{\gamma_1 V^3 K}{2\lambda} \right) + \frac{A^2 \lambda \pi^2 Sgn\beta}{2K^2 Sinh \frac{\pi K'}{2K}} \left(\frac{2\pi^2 i V \gamma_1 c}{\lambda^2} - c_t \right)$$
$$+ \frac{4A^2 \pi^2 c^2}{\lambda^2} \left(3V\gamma_1 - \frac{\beta_1 \lambda}{2K} \right) \left(c_1 - \frac{\pi^2}{K Sinh \frac{\pi K'}{2K}} \right) + \frac{16 A^2 K c_2}{3\lambda} \left(V\gamma_1 - \beta_1 - \frac{V}{2} \right) \tag{26}$$
$$+ \frac{A^2 \lambda c_3}{3K} \left(\frac{\delta_1}{2} - \frac{3\alpha_1 V}{2} \right) - \frac{A^2 V \gamma_1 K}{\lambda} \left[\frac{d}{dz} (dn + \beta cn)^2 \right]_{-2k}^{2k}$$

Integrated variation ϕ equation:

$$A^2 \left[\frac{2E}{K} - (1-\beta^2) \right] = \frac{N}{\lambda} \tag{27}$$

Variation c equation:

$$\frac{dQ}{dt} + \frac{CK}{\lambda M} \left(6V\gamma_1 - \frac{\beta_1 \lambda}{K} \right) = 0 \tag{28}$$

 ISSN: 2454-1907
DOI: 10.5281/zenodo.1149113

Where Q is given by

$$Q = \frac{\pi^2 \lambda A^2 \operatorname{sgn}\beta}{2K^2 \sinh \frac{\pi K'}{2K}} \quad \text{And the time-dependent mass } M = \frac{\lambda}{4\pi^2}\left(\frac{N}{\lambda} - \frac{A^2\pi^2}{K^2 \sinh \frac{\pi K'}{2K}}\right)^{-1}$$

The Hamiltonian H:

$$H = \frac{Nv^2}{2} - N\phi_t - Qc_t - A^2\left[\alpha^{-1}\left(\frac{Vx_{0t}}{2} - \frac{\beta_1 V^2}{4} - \phi_t\right) + \frac{\gamma_1 V^3}{8}\right]I_2 - A^2\alpha^{-1}\left(-c_t + \frac{2\pi^2 i\gamma_1 Vc}{\lambda^2}\right)I_1 - \frac{2A^2\pi^2 c^2}{\lambda^2}\left(3V\gamma_1 - 2\beta_1\alpha^{-1}\right)I_3 - A^2\alpha(\gamma_1 V - \beta_1)I_4 - A^4\alpha^{-1}\left(\frac{\delta_1}{2} - \frac{3\alpha_1 V}{2}\right)I_5 + \frac{\gamma_1 A^2 \alpha V}{2} I_{12} \tag{29}$$

Variation A equation:

$$N\phi_t + Qc_t = N\frac{V^2}{2}(1-\beta_1) + \frac{2\pi^2 ci\gamma_1 Q}{\lambda^2} + \frac{c^2 K}{\lambda M}\left(3V\gamma_1 - \frac{\beta_1\lambda}{K}\right) + \frac{16Kc_2 A^2}{3\lambda}\left(V\gamma_1 - \beta - \frac{V}{2}\right) + \frac{A^4\lambda c_3}{3K}(\delta - 3\alpha_1 V) - \frac{A^2 V\gamma_1 K}{\lambda}\left[\frac{d}{dz}(dn + \beta cn)^2\right]_{-2k}^{2k} \tag{30}$$

A simple qualitative analysis is also possible as a result of the Hamiltonian structure of the equations (27) to (30). It can be shown that the "mass" M of equation is positive and finite for all $|\beta|$ □□1,while the generalized coordinate Q is an increasing function of β, continuously differentiable for $\beta \in [-1,1]$ including β =0.

3. Conclusion

We applied Ritz variational method to construct a model function between linear and non-linear evolution of modulational instability. Spatial variance of trial function was assumed a priori while time dependence of its parameters was subject to optimization. We choose travelling solutions in the form of a combination of Jacobi Elliptic functions (with an appropriate phase factor). The periodicity length and group velocities were assumed constant. If the limit $\alpha_1 = \gamma_1 = 0,$ the result is same as calculated in [14].

References

[1] E. Kit and L. Shemer (2002) J. Fluid Mech. 450, 201–205.
[2] D. Anderson (1983) Phys. Rev. A 27 (6), 3135–3145.
[3] N. J. Zabusky and M. D. Kruskal (1965) Phys. Rev. Lett. 15(6), 240–243.
[4] H. Airault, H. P. McKean, and J. Moser,Comm. (1977) PureAppl. Math. 30(1), 95–148.

ISSN: 2454-1907
DOI: 10.5281/zenodo.1149113

[5] E.R.Tracy, H.H.Chen and Y.C. Lee (1984) Phys. Rev. Lett. 53, 218 – 221.
[6] B. Hafizi (1981) Phys.Fluids 24(10), 1791–1798.
[7] Q. S. Chang, B. L. Guo, and H. Jiang (1995) Math. Comp. 64(210), 537–553, S7–S11.
[8] A. O. Smirnov (1989) Mat.Zametki 45(6), 66–73, 111 (Russian).
[9] M. Sigal, Comm. (1993) Math. Phys. 153(2), 297–320.
[10] P. L. Christiansen, J. C. Eilbeck, V. Z. Enolskii, and N. A. Kostov, (1995)Proc. Roy.Soc. London Ser. A 451(1943), 685–700.
[11] A.V. Porubov and D. F. Parker, (1999) Wave Motion 29(2), 97–109.
[12] F. F. Sun, (2003), Master's thesis, National University, Singapore.
[13] V. E. Zakharov and A. B. Shabat, Ž.Ėksper. (1971)Teoret. Fiz 61(1), 118–134, (1972) Soviet Physics JETP 34(1), 62–69.
[14] Arun Kumar, (2009), International Journal of Computational and Applied Mathematics ISSN 1819-4966, 4(2) 159–164.
[15] LIANG Zu-Feng, TANG Xiao-Yan,(2010) CHIN. PHYS. LETT. Vol. 27(3)030201.
[16] Hirota R (1973) J. Math Phys. 14 805.
[17] Jiao X Y and Lou S Y (2009) Chin. Phys. Lett. 26 040202.
[18] Zhao S L, Zhang D J and Chen D Y (2009) Chin. Phys. Lett.26 030202.
[19] Gao Y, Tang X Y and Lou S Y (2009) Chin. Phys. Lett. 26 030502
[20] Yang J R and Mao J J (2008) Chin. Phys. Lett. 25 1527
[21] Yan T, Yu J L and Huang N N (2008) Chin. Phys. Lett. 2552
[22] Tang X Y, Lou S Y and Zhang Y (2002) Phys.Rev.E 66046601
[23] Tang X Y and Lou S Y (2003)J. Math. Phys. 44 4000
[24] Tang X Y and Lou S Y (2009) Front. Phys. Chin. 2 235
[25] Tang X Y and Liang Z F (2006) Phys. Lett.A 351 398

*Corresponding author.
E-mail address: drrdpankaj@ yahoo.com